

# Multiquark picture for $\Lambda(1405)$ and $\Sigma(1620)$

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**Abstract.** We propose a new QCD sum rule analysis for the  $\Lambda(1405)$  and the  $\Sigma(1620)$ . Using the  $I=0$  and  $I=1$  multiquark sum rules we predict their masses.

One of interesting subjects in nuclear physics is to study properties of the excited baryon states. For example, in the case of the  $\Lambda(1405)$  its nature is not revealed completely [1]; i.e. an ordinary three quark state or a  $\bar{K}N$  bound state or the mixing state of the previous two possibilities. In the QCD sum rule approach [2] there have been several works on the  $\Lambda(1405)$  using three-quark interpolating fields [3,4] or five-quark operators [5]. In this work we focus on the decay modes of the  $\Lambda(1405)$  and the  $\Sigma(1620)$  and get the mass of each particle introducing multiquark sum rules.

Let's consider the following correlator:

$$\Pi(q^2) = i \int d^4x e^{iqx} \langle T(J(x) \bar{J}(0)) \rangle, \quad (1)$$

where  $J$  is the  $\pi\Sigma$  ( $I=0$ ) multiquark interpolating field,  $J_{\pi^+\Sigma^- + \pi^0\Sigma^0 + \pi^-\Sigma^+}$ .

Here, for the  $\Sigma$  we take the Ioffe's choice [6]; e.g.  $\pi^0\Sigma^0$  means  $\epsilon_{abc}(\bar{u}_e i\gamma^5 u_e - \bar{d}_e i\gamma^5 d_e)([u_a^T C \gamma_\mu s_b] \gamma^5 \gamma^\mu d_c + [d_a^T C \gamma_\mu s_b] \gamma^5 \gamma^\mu u_c)$ , where  $u, d$  and  $s$  are the up, down and strange quark fields, and  $a, b, c, e$  are color indices.  $T$  denotes the transpose in Dirac space and  $C$  is the charge conjugation matrix.

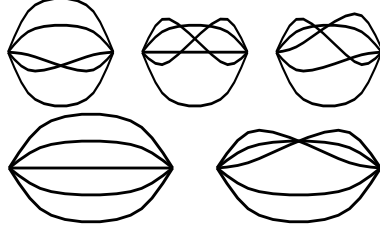
The OPE side has two structures:

$$\Pi^{OPE}(q^2) = \Pi_q^{OPE}(q^2) \not{q} + \Pi_1^{OPE}(q^2) \mathbf{1}. \quad (2)$$

In this paper, however, we only present the sum rule from the  $\Pi_1$  structure (hereafter referred to as the  $\Pi_1$  sum rule) because the  $\Pi_1$  sum rule is generally more reliable than the  $\Pi_q$  sum rule as emphasized in Ref. [7]. The OPE side is given as follows.

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**FIGURE 1.** “bound” (upper) and “unbound” (lower) diagrams. Solid lines are the quark propagators.

$$\begin{aligned}
\Pi_1^{OPE}(q^2) = & -\frac{7}{\pi^8} \frac{m_s}{2^{18} 3^2} q^{10} \ln(-q^2) + \frac{7}{\pi^6} \frac{1}{2^{15} 3^2} \langle \bar{s}s \rangle q^8 \ln(-q^2) \\
& + \frac{35}{\pi^6} \frac{m_s^2}{2^{14} 3^2} \langle \bar{s}s \rangle q^6 \ln(-q^2) - \frac{121}{\pi^4} \frac{m_s}{2^9 3^2} \langle \bar{q}q \rangle^2 q^4 \ln(-q^2) \\
& + \frac{11}{\pi^2} \frac{1}{2^6} \langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle q^2 \ln(-q^2) - \frac{m_s^2}{\pi^2} \frac{1}{2^6} \frac{1}{3} (14 \langle \bar{q}q \rangle^3 - 33 \langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle) \ln(-q^2) \\
& - \frac{m_s}{2^4} \frac{1}{3^3} (140 \langle \bar{q}q \rangle^4 + 3 \langle \bar{q}q \rangle^3 \langle \bar{s}s \rangle) \frac{1}{q^2}, \tag{3}
\end{aligned}$$

where  $m_s$  is the strange quark mass and  $\langle \bar{q}q \rangle$ ,  $\langle \bar{s}s \rangle$  are the quark condensate and the strange quark condensate, respectively. Here, we let  $m_u = m_d = 0 \neq m_s$  and  $\langle \bar{u}u \rangle = \langle \bar{d}d \rangle \equiv \langle \bar{q}q \rangle \neq \langle \bar{s}s \rangle$ . We neglect the contribution of gluon condensates and concentrate on tree diagrams such as Fig. 1, and assume the vacuum saturation hypothesis to calculate quark condensates of higher dimensions. Note that only some typical diagrams are shown in Fig. 1.

The contribution of the “bound” diagrams is a  $1/N_c$  correction to that of the “unbound” diagrams, where  $N_c$  is the number of the colors. In Eq. (3) we set  $N_c = 3$ . The “unbound” diagrams correspond to a picture that two particles are flying away without any interaction between them. In the  $N_c \rightarrow \infty$  limit only the “unbound” diagrams contribute to the  $\pi\Sigma$  multiquark sum rule. Then, the  $\pi\Sigma$  multiquark mass ( $m(\pi\Sigma)$ ) should be the sum of the pion and the  $\Sigma$  mass in this limit.

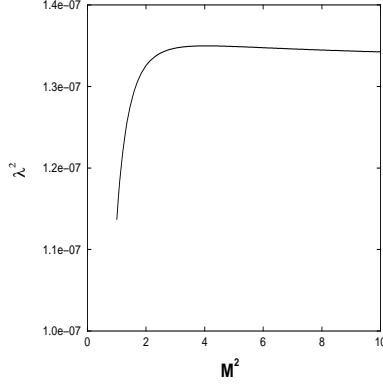
Eq. (3) has the following form:

$$\begin{aligned}
\Pi_1^{OPE}(q^2) = & a q^{10} \ln(-q^2) + b q^8 \ln(-q^2) + c q^6 \ln(-q^2) + d q^4 \ln(-q^2) \\
& + e q^2 \ln(-q^2) + f \ln(-q^2) + g \frac{1}{q^2}, \tag{4}
\end{aligned}$$

where  $a, b, c, \dots, g$  are constants. Then, we parameterize the phenomenological side as

$$\frac{1}{\pi} \text{Im} \Pi_1^{Phen}(s) = \lambda^2 m \delta(s - m^2) + [-a s^5 - b s^4 - c s^3 - d s^2 - e s - f] \theta(s - s_0), \tag{5}$$

where  $m$  is the  $m(\pi\Sigma)$  and  $s_0$  the continuum threshold.  $\lambda$  is the coupling strength of the interpolating field to the physical  $\Lambda$  (1405) state. The Borel-mass dependence



**FIGURE 2.** The Borel-mass dependence of the coupling strength  $\lambda^2$  from the  $\pi\Sigma$  multiquark sum rule at  $s_0 = 2.789 \text{ GeV}^2$ .

of the  $m(\pi\Sigma)$  shows that there is a plateau for the large Borel mass. However, this is a trivial result from our crude model on the phenomenological side. Hence we do not take this as the  $m(\pi\Sigma)$  and neither as the  $\Lambda(1405)$  mass. Instead, we draw the Borel-mass dependence of the coupling strength  $\lambda^2$  at  $s_0 = 2.789 \text{ GeV}^2$  as shown in Fig. 2, where the  $s_0$  is taken by considering the next  $\Lambda$  particle [1]. There is the maximum point in the figure. It means that the  $\pi\Sigma$  multiquark state couples strongly to the physical  $\Lambda(1405)$  state at this point. Then we take the  $\Lambda(1405)$  mass as the  $m(\pi\Sigma)$  at the point. However, it would be better to determine an effective threshold  $s_0$  from the present sum rule itself.

Thus, the steps for getting the  $m(\pi\Sigma)$  are as follows. First, consider “unbound” diagrams only and choose a threshold  $s_0$  in order that the average mass between the fiducial Borel interval becomes the  $m(\pi) + m(\Sigma)$ . Second, consider whole diagrams (“unbound” + “bound” diagrams) and draw the Borel-mass dependence of the coupling strength  $\lambda^2$  using the above  $s_0$ . Last, determine the  $m(\pi\Sigma)$  where the  $\lambda^2$  has the maximum value, and thus take this as the  $\Lambda(1405)$  mass. Following the above steps we get the  $m(\pi\Sigma) = 1.424 \text{ GeV}$  at  $s_0 = 3.082 \text{ GeV}^2$ .

There is another  $I=0$  multiquark state; i.e. the  $\bar{K}^0 n + K^- p$  multiquark state. Similarly, we obtain the  $m(\bar{K}N) = 1.589 \text{ GeV}$  at  $s_0 = 3.852 \text{ GeV}^2$ . This corresponds to the  $\Lambda(1600)$  mass. It is interesting to note that the masses from two multiquark states are similar at the same threshold as shown in Table 1.

Now, we can extend our previous analysis to the  $I=1$  multiquark states and thus get the  $\Sigma(1620)$  mass. There are three decay channels for the  $\Sigma(1620)$ . Then, we

**TABLE 1.** Mass of the  $\bar{K}N$ ,  $\pi\Sigma$  ( $I=0$ ) multiquark states ( $\langle \bar{q}q \rangle = -(0.230 \text{ GeV})^3$ ,  $\langle \bar{s}s \rangle = 0.8 \langle \bar{q}q \rangle$ , and  $m_s = 0.150 \text{ GeV}$ ).

$s_0 \text{ (GeV}^2\text{)}$	$m(\bar{K}N) \text{ (GeV)}$	$m(\pi\Sigma) \text{ (GeV)}$
3.852	1.589	1.612
3.082	1.405	1.424

**TABLE 2.** Mass of the  $\bar{K}N$ ,  $\pi\Sigma$ , and  $\pi\Lambda$  (I=1) multiquark states ( $\langle\bar{q}q\rangle = - (0.230 \text{ GeV})^3$ ,  $\langle\bar{s}s\rangle = 0.8 \langle\bar{q}q\rangle$ , and  $m_s = 0.150 \text{ GeV}$ ).

$s_0 \text{ (GeV}^2\text{)}$	$m(\bar{K}N) \text{ (GeV)}$	$m(\pi\Sigma) \text{ (GeV)}$	$m(\pi\Lambda) \text{ (GeV)}$
3.852	1.589	1.606	1.581

can construct the following multiquark interpolating fields;  $J_{\bar{K}^0 n - K^- p}$ ,  $J_{\pi^+ \Sigma^- - \pi^- \Sigma^+}$ , and  $J_{\pi^0 \Lambda}$  ( or  $J_{\pi^\pm \Lambda}$ ). In Table 2 we present each multiquark mass.

We have obtained the I=0 and I=1 multiquark masses which are slightly different from the experimental values [1]. One of corrections is to include the isospin symmetry breaking effects (i.e.  $m_u \neq m_d \neq 0$ ,  $\langle\bar{u}u\rangle \neq \langle\bar{d}d\rangle$ , and electromagnetic effects) in our sum rules. On the other hand, one can consider the contractions between the  $\bar{u}$  and  $u$  (or between the  $\bar{d}$  and  $d$ ) quarks in the initial state which have been excluded in our previous calculation. However, it is found that this correction is very small comparing to other  $1/N_c$  corrections, i.e. the contribution of “bound” diagrams. Another possibility is the correction from the possible instanton effects [8] to the I=0 and I=1 states, respectively.

In this work we have neglected the contribution of gluon condensates and that of other higher dimensional operators including gluon components. Since we have considered the  $\Pi_1$  sum rule, only the odd dimensional operators can contribute to the sum rule. Thus, for example, the contribution of the gluon condensates is given by the terms like  $m_s \langle \frac{\alpha_s}{\pi} G^2 \rangle$  and thus can be neglected comparing to other quark condensates of the same dimension.

In summary, the  $\Lambda$  (1405) and  $\Sigma$  (1620) masses are predicted in the QCD sum rule approach using the  $\bar{K}N$ ,  $\pi\Sigma$ , and  $\pi\Lambda$  multiquark interpolating fields (both I=0 and I=1).

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